Name ______________________ Class ____________ Date ________

17.1 Understanding Polynomial Expressions

Resource Locker

Essential Question: What are polynomial expressions, and how do you simplify them?

Explore Identifying Monomials

A **monomial** is an expression consisting of a number, variable, or product of numbers and variables that have whole number exponents. *Terms* of an expression are parts of the expression separated by plus signs. (Remember that $x - y$ can be written as $x + (-y)$.) A monomial cannot have more than one term, and it cannot have a variable in its denominator. Here are some examples of monomials and expressions that are not monomials.

Monomials					Not Monomials				
4	x	$-4xy$	$0.25x^3$	$\frac{xy}{4}$	$4 + x$	$x - 1$	$0.7x^{-2}$	$0.25x^{-1}$	$\frac{y}{x^3}$

Use the following process to determine if $5ab^2$ is a monomial.

(A) $5ab^2$ has ________ term(s), so it ________ be a monomial.

(B) Does $5ab^2$ have a denominator?

____

(C) If possible, split it into a product of numbers and variables.

$5ab^2 = 5 \cdot \square \cdot \square$

(D) List the numbers and variables in the product.

Numbers: $\square$ Variables: $\square$

(E) Check the exponent of each variable. Complete the following table.

Variable	Exponent
a	
b	

(F) The exponents of the variables in $5ab^2$ are all ________________.
Therefore, $5ab^2$ _____ a monomial.

(G) Is $\frac{5}{k^2}$ a monomial?

____

Ⓗ Complete the table below.

Term	Is this a monomial?	Explain your reasoning.
$5ab^2$	yes	$5ab^2$ is the product of a number, 5, and the variables a and b.
x^2		
$\sqrt{y}$	no	
2^2		
$\frac{5}{k^2}$	no	
$5x + 7$		
$x^2 + 4ab$		
$\frac{k^2}{4}$		

Reflect

1. **Discussion** Explain why $16^{\frac{1}{3}}$ is a monomial but $x^{\frac{1}{3}}$ is not a monomial.

2. **Discussion** Is x^0 a monomial? Justify your answer in two ways.

Explain 1 Classifying Polynomials

A **polynomial** can be a monomial or the sum of monomials. Polynomials are classified by the number of terms they contain. A monomial has one term, a **binomial** has two terms, and a **trinomial** has three terms. $8xy^2 - 5x^3y^3z$, for example, is a binomial.

Polynomials are also classified by their degree. The **degree of a polynomial** is the greatest value among the sums of the exponents on the variables in each term.

The binomial $8xy^2 - 5x^3y^3z$ has two terms. The variables in the first term are x and y. The exponent on x is 1, and the exponent on y is 2. The number 8 is not a variable, so it has a degree of 0. The first term has a degree of $0 + 1 + 2 = 3$. The degree of the second term is $0 + 3 + 3 + 1 = 7$. Therefore, $8xy^2 - 5x^3y^3z$ is a 7^{th} degree binomial.

Example 1 **Classify each polynomial by its degree and the number of terms.**

(A) $7x^2 - 5x^3y^3$

Find the degree of each term by adding the exponents of the variables in that term. The greatest degree is the degree of the polynomial. The degree of the term $-5x^3y^3$ is 6, which you obtain by adding the exponents of x and y: $6 = 3 + 3$. Numbers have degree 0.

$7x^2 - 5x^3y^3$

Degree : 6	$7x^2$ has degree 2, and $-5x^3y^3$ has degree $6 = 3 + 3$.
Binomial	There are two terms.

(B) $3^2 + 2n^3 + 8n$

$3^2 + 2n^3 + 8n$

Degree: $\boxed{}$	3^2 has degree $\boxed{}$, $2n^3$ has degree $\boxed{}$, and $8n$ has degree $\boxed{}$.
Trinomial	There are $\boxed{}$ terms.

Reflect

3. What is the degree of $5x^0y^0 + 5$?

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4. Is $5x^0y^{0.5} + 5$ a polynomial? Justify your answer.

Your Turn

Classify each polynomial by its degree and the number of terms.

5. $3x^2y^2 + 3xy^2 + 5xy$

6. $8ab^2 - 3a^2b$

Explain 2 Writing Polynomials in Standard Form

The terms of a polynomial may be written in any order, but when a polynomial contains only one variable there is a standard form in which it can be written.

The **standard form of a polynomial** containing only one variable is written with the terms in order of decreasing degree. The first term will have the greatest degree, the next term will have the next greatest degree, and so on, until the final term, which will have the lowest degree.

When written in this form, the coefficient of the first term is called **the leading coefficient**.

$5x^4 + 4x^2 + x - 2$ is a 4^{th} degree polynomial written in standard form. It consists of one variable, and its first term is $5x^4$. The leading coefficient is 5 because it is in front of the highest-degree term.

Example 2 **Write each polynomial in standard form. Then give the leading coefficient.**

(A) $20x - 4x^3 + 1 - 2x^2$

Find the degree of each term and then arrange them in descending order of their degree.

$$\text{Degree:}\quad \underbrace{20x}_{1}\ \underbrace{-4x^3}_{3}\ \underbrace{+\ 1}_{0}\ \underbrace{-2x^2}_{2} \ = \ \underbrace{-4x^3}_{3}\ \underbrace{-2x^2}_{2}\ \underbrace{+\ 20x}_{1}\ \underbrace{+\ 1}_{0}$$

The standard form is $-4x^3 - 2x^2 + 20x + 1$. The leading coefficient is -4.

(B) $z^3 - z^6 + 4z$

Find the degree of each term and then arrange them in descending order of their degree.

$$\text{Degree:}\quad \underbrace{z^3}_{\square}\ \underbrace{-z^6}_{\square}\ \underbrace{+4z}_{\square} = \underbrace{\square}_{\square}\ \underbrace{\square}_{\square}\ \underbrace{\square}_{\square}$$

The standard form is ______________. The leading coefficient is $\square$.

Your Turn

Write each polynomial in standard form. Then give the leading coefficient.

7. $10 - 3x^2 + x^5 + 4x^3$

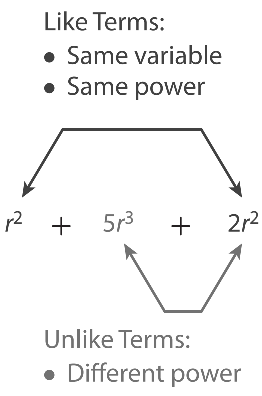
8. $18y^5 - 3y^8 + 10y$

9. $10x + 13 - 15x^2$

10. $-3b^2 + 2b - 7 + 6b^3 + 12b^4 + 7$

Explain 3 Simplifying Polynomials

Polynomials are simplified by combining like terms. Like terms are monomials that have the same variables raised to the same powers. Unlike terms have different powers.



Identify like terms and combine them using the Distributive Property. Simplify.

$r^2 + 5r^3 + 2r^2$

$(r^2 + 2r^2) + 5r^3$ Identify like terms by grouping them together in parentheses.

$r^2(1 + 2) + 5r^3$ Combine using the Distributive Property.

$3r^2 + 5r^3$ Simplify.

Example 3 **Combine like terms to simplify each polynomial.**

(A) $-2y^3 - 8y^2 + y^2 + 2y^3$

$-2y^3 + 2y^3 - 8y^2 + y^2$ Rearrange in descending order of exponents.

$(-2y^3 + 2y^3) + (-8y^2 + y^2)$ Group like terms.

$y^3(-2 + 2) + y^2(-8 + 1)$ Combine using the Distributive Property.

$y^3(0) + y^2(-7)$ Simplify.

$-7y^2$

(B) $p^2q^3 - 4p^5q^4 - 4p^2q^3 + 3p^5q^4$

________________ Rearrange in descending order of exponents.

________________ Group like terms.

$p^5q^4(\boxed{}) + p^2q^3(\boxed{})$ Combine using the Distributive Property.

$p^5q^4(\boxed{}) + p^2q^3(\boxed{})$ Simplify.

____

Reflect

11. Can you combine like terms without formally showing the Distributive Property? Explain.

Your Turn

Simplify.

12. $3p^2q^2 - 3p^2q^3 + 4p^2q^3 - 3p^2q^2 + pq$

13. $3(a + b) - 6(b + c) + 8(a - c)$

14. $ab - a^2 + 4^2 - 5ab + 3a^2 + 10$

Explain 4 Evaluating Polynomials

Given a polynomial expression describing a real-world situation and a specific value for the variable(s), evaluate the polynomial by substituting for the variable(s). Then interpret the result.

Example 4 **Evaluate the given polynomial to find the solution in each real-world scenario.**

(A) A skyrocket is launched from a 6-foot-high platform with an initial speed of 200 feet per second. The polynomial $-16t^2 + 200t + 6$ gives the height in feet that the skyrocket will rise in t seconds. How high will the rocket rise if it has a 5-second fuse?

$-16t^2 + 200t + 6$	Write the expression.
$-16(5)^2 + 200(5) + 6$	Substitute 5 for t.
$-16(25) + 200(5) + 6$	Simplify using the order of operations.
$-400 + 1000 + 6$	
606	

The rocket will rise 606 feet.

(B) Lisa wants to measure the depth of an empty well. She drops a ball from a height of 3 feet into the well and measures how long it takes the ball to hit the bottom of the well. She uses a stopwatch, starting when she lets go of the ball and ending when she hears the ball hit the bottom of the well. The polynomial $-16t^2 + 0t + 3$ gives the height of the ball after t seconds where 0 is the initial speed of the ball and 3 is the initial height the ball was dropped from. Her stopwatch measured a time of 2.2 seconds. How deep is the well? (Neglect the speed of sound and air resistance).

______________	Write the expression.
______________	Substitute ______ for t.
______________	Simplify using order of operations.
______________	

The ball goes ______ feet below the ground. The well is ______ feet deep.

Your Turn

Solve each real-world scenario.

15. Nate's client said she wanted the width w of every room in her house increased by 2 feet and the length $2w$ decreased by 5 feet. The polynomial $(2w - 5)(w + 2)$ or $2w^2 - w - 10$ gives the new area of any room in the house. The current width of the kitchen is 16 feet. What is the area of the new kitchen?

16. A skyrocket is launched from a 20-foot-high platform, with an initial speed of 200 feet per second. If the polynomial $-16t^2 + 200t + 20$ gives the height that the rocket will rise in t seconds, how high will a rocket with a 4-second fuse rise?

Elaborate

17. What is the degree of the expression $-16t^2 + 200t + 20$, where t is a variable? What is the degree of the expression if $t = 1$? Are the expressions monomials, binomials, or trinomials?

18. Two cars drive toward each other along a straight road at a constant speed. The distance between the cars is $\ell - (r_1 + r_2)t$, where r_1 and r_2 are their speeds, t is a variable representing time and ℓ is the length of their original separation. Write the expression in standard form. What is its degree? What is its leading coefficient?

19. The polynomial $-16t^2 + 200t + 20$ gives the height of a projectile launched with an initial speed of 200 feet per second t seconds after launch. A second projectile is launched at the same time but with an initial speed of 300 feet per second, with its height given by the polynomial $-16t^2 + 300t + 20$. How much higher will the second projectile be than the first after 10 seconds?

20. **Essential Question Check-In** What do you have to do to simplify sums of polynomials? What property do you use to accomplish this?

Evaluate: Homework and Practice

Personal Math Trainer

- Online Homework
- Hints and Help
- Extra Practice

1. Is $(5 + 4x^0)2x$ a monomial? What about $(5 + 4x^2)2x$?

2. Is the sum of two monomials always a monomial? Is their product always a monomial?

Classify each polynomial by its degree and the number of terms.

3. $x^2 - 5x^3$

4. $x^2 - x^4 + y^2x^3$

5. $a^4b^3 - a^3b^2 + a^2b$

6. $15 + x\sqrt{2}$

7. $x + y + z$

8. $a^5 + b^2 + a^2b^2$

Write each polynomial in standard form. Then give the leading coefficient.

9. $2x - 40x^3 - 2x^2$

10. $3 + c - c^2$

11. $3b^2 - 2b + b^2$

12. $4a - 3a + 21 + 6$

Simplify each polynomial.

13. $-2y^3 - y^2 + y^2 + y^3$

14. $-y^3x - y^2x + y^2 + y^3x + y^2$

15. $xyz\sqrt[3]{2} + 2^5xyz + 2^{10}xy$

16. $a^3 + a^2 + ab$

Use the information to solve the problem.

17. **Persevere in Problem Solving** Lisa is measuring the depth of a well. She drops a ball from a height of h feet into the well and measures how long it takes the ball to hit the bottom of the well. The polynomial $-16t^2 + 0t + h$ models this situation, where 0 is the initial speed of the ball and h is the height it was dropped from. (This is a different well from the problem you solved before.) She raises her arm very high and drops the ball from a height of 6.0 feet. Her stopwatch measured a time of 3.5 seconds. How deep is the well?

18. **Multi-Step** Claire and Richard are both artists who use square canvases. Claire uses the polynomial $50x^2 + 250$ to decide how much to charge for her paintings, and Richard uses the polynomial $40x^2 + 350$ to decide how much to charge for his paintings. In each polynomial, x is the height of the painting in feet.

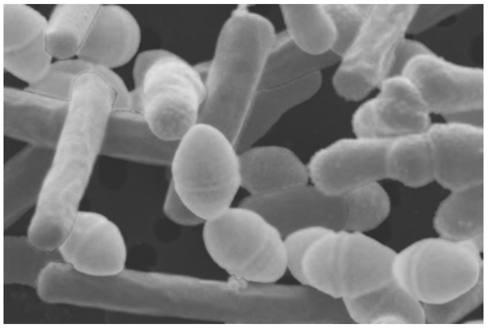
a. How much does Claire charge for a 6-foot painting?

b. How much does Richard charge for a 5-foot painting?

c. To the nearest tenth, for what height will both Claire and Richard charge the same amount for a painting? Explain how to find the answer.

d. When both Claire and Richard charge the same amount for a painting, how much does each charge?

19. Make a Prediction The number of cells in a bacteria colony increases according to a polynomial expression that depends on the temperature. The expression for the number of bacteria is $t^2 + 4t + 4$ when the temperature of the colony is 20°C and $t^2 + 3t + 4$ when the colony grows at 30 °C. t represents the time in seconds that the colony grows at the given temperature.



a. After 1 minute, will the population be greater in a colony at 20 °C or 30 °C? Explain.

b. After 10 minutes, how will the colonies compare in size? Explain.

c. After 1000 minutes, how will the colonies compare in size? Will one colony always have more bacteria? Explain.

20. Two cars are driving toward each other along a straight road. Their separation distance is $\ell - (r_1 + r_2)t$, where ℓ is their original separation distance and r_1 and r_2 are their speeds. Will the cars meet? When? What if they are going in the same direction and not driving toward one another? Will they meet then?

H.O.T. **Focus on Higher Order Thinking**

21. Explain the Error Enrique thinks that the polynomial $2^2x^2 + 2^3x + 2^4$ has a degree of 4 since $2 + 2 = 3 + 1 = 4$. Explain his error and determine the correct degree.

22. Analyze Relationships Sewell is doing a problem regarding the area of pairs of squares. Sewell says that the expression $(x + 1)^2$ will be greater than $(x - 1)^2$ for all values of x because $x + 1$ will always be greater than $x - 1$. Why is he correct when the expressions are areas of squares? Is he correct for any real x outside this model?

23. Counterexamples Prove by counterexample that the sum of monomials is not necessarily a monomial.

24. Communicate Mathematical Ideas Polynomials are simplified by combining like terms. When combining like terms, you use the Distributive Property. Prove that the Distributive Property, $a \cdot (b + c) = a \cdot b + a \cdot c$, holds over the positive integers $a, b, c > 0$ from the definition of multiplication: $a \cdot b = \underbrace{a + a + \ldots + a}_{b \text{ times}}$.

25. Analyze Relationships A right triangle has height h and base $h + 8$. Write an expression that represents the area of the triangle. Then calculate the area of a triangle with a height of 16 cm.

Lesson Performance Task

A pyrotechnics specialist is designing a firework spectacular for a company's 75th anniversary celebration. She can vary the launch speed to 200, 250, 300, or 400 feet per second, and can set the fuse on each firework for 3, 4, 5, or 6 seconds. Create a table of the various heights the fireworks can explode at if the height of the firework is modeled by the function $h(t) = -16t^2 + v_0 t$, where t is the time in seconds and v_0 is the initial speed of the firework.

Design a fireworks show using 3 firing heights and at least 30 fireworks. Have some fireworks go off simultaneously at different heights. Describe your display so you will know what needs to be launched and when they will go off.